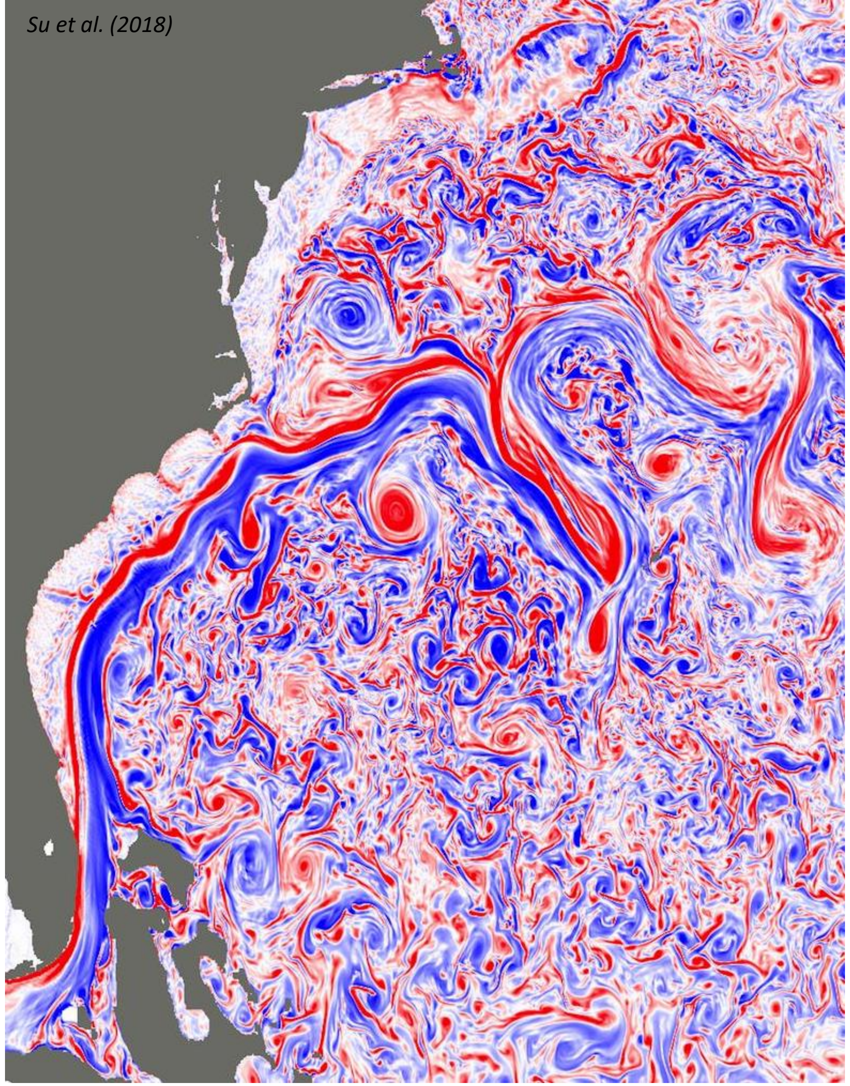
Image credit: 66 North

Su et al. (2018)



"Southern Ocean" generated by https://app.wombo.art/

Spurious forces can dominate the vorticity budget of ocean gyres on the C-grid

Andrew Styles[1], Mike Bell[2], David Marshall[1], David Storkey[2]

[1]Department of Physics | University of Oxford

[2]Met Office

The vorticity budget

A vorticity budget emerges from the depth-integrated vorticity equation:

$$\frac{\partial \bar{\zeta}}{\partial t} = \underbrace{-\boldsymbol{\nabla_h} \cdot (f\boldsymbol{U})}_{\text{Coriolis term}} + \underbrace{\frac{1}{\rho_0}(\boldsymbol{\nabla} P_b \times \boldsymbol{\nabla} H) \cdot \hat{\boldsymbol{k}}}_{\text{Bottom pressure torque}} + \underbrace{\frac{1}{\rho_0}\boldsymbol{\nabla} \times \boldsymbol{\tau_{wind}} \cdot \hat{\boldsymbol{k}}}_{\text{Wind stress curl}} - \underbrace{\frac{1}{\rho_0}\boldsymbol{\nabla} \times \boldsymbol{\tau_{bot}} \cdot \hat{\boldsymbol{k}}}_{\text{Bottom stress curl}} + \underbrace{A^{\zeta}}_{\text{Non-linear advection of vorticity}} + \underbrace{D^{\zeta}}_{\text{Lateral diffusion}}$$

Coriolis term $\approx \beta V$ for steady and incompressible flow

Bottom **pressure** torque

Wind stress curl

Bottom **stress** curl

Non-linear **advection** of vorticity

Lateral **diffusion**

The vorticity budget

A term in this vorticity equation describes **the rotational effect** of a **depth-integrated acceleration**

$$\underbrace{\Omega}_{\textbf{Vorticity term}} = \nabla \times \left(\int_{-H}^{\eta} \underbrace{\boldsymbol{a}}_{\textbf{Acceleration}} \, dz \right) \cdot \hat{\boldsymbol{k}}$$

The same calculation method can be applied to **discrete accelerations** in **ocean models**

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The vorticity budget

A term in this vorticity equation describes **the rotational effect** of a **depth-integrated acceleration**

$$\underbrace{-\boldsymbol{\nabla}_{\boldsymbol{h}} \cdot (f\boldsymbol{U})}_{\text{Coriolis vorticity term}} = \nabla \times \left(\int_{-H}^{\eta} \underbrace{- f\widehat{\boldsymbol{k}} \times \boldsymbol{u}}_{\text{Coriolis acceleration}} \, dz \right) \cdot \widehat{\boldsymbol{k}}$$

Coriolis vorticity term

Coriolis acceleration

The same calculation method can be applied to **discrete accelerations** in **ocean models**

The vorticity budget

This is one of **many ways** to formulate a **vorticity budget:**

$$\boldsymbol{\nabla} \times \left(\int_{-H}^{\eta} \boldsymbol{a}\, dz \right) \cdot \widehat{\boldsymbol{k}}$$

$$\boldsymbol{\nabla} \times \left(\frac{1}{H} \int_{-H}^{\eta} \boldsymbol{a}\, dz \right) \cdot \widehat{\boldsymbol{k}}$$

$$\left(\int_{-H}^{\eta} \widehat{\boldsymbol{k}} \cdot (\nabla \times \boldsymbol{a}) \;\, dz \right) \cdot \widehat{\boldsymbol{k}}$$

$$\boldsymbol{\nabla} \times \left(\frac{1}{f} \int_{-H}^{\eta} \boldsymbol{a}\, dz \right) \cdot \widehat{\boldsymbol{k}}$$

All four are **physically distinct** budgets with different **contributions**

The vorticity budget

This is one of **many ways** to formulate a **vorticity budget:**

$$\nabla \times \left(\int_{-H}^{\eta} \boldsymbol{a} \, dz \right) \cdot \widehat{\boldsymbol{k}}$$

We use this one

$$\nabla \times \left(\frac{1}{H} \int_{-H}^{\eta} \boldsymbol{a} \, dz \right) \cdot \widehat{\boldsymbol{k}}$$

$$\left(\int_{-H}^{\eta} \widehat{\boldsymbol{k}} \cdot (\nabla \times \boldsymbol{a}) \; dz \right) \cdot \widehat{\boldsymbol{k}}$$

$$\nabla \times \left(\frac{1}{f} \int_{-H}^{\eta} \boldsymbol{a} \, dz \right) \cdot \widehat{\boldsymbol{k}}$$

All four are **physically distinct** budgets with different **contributions**

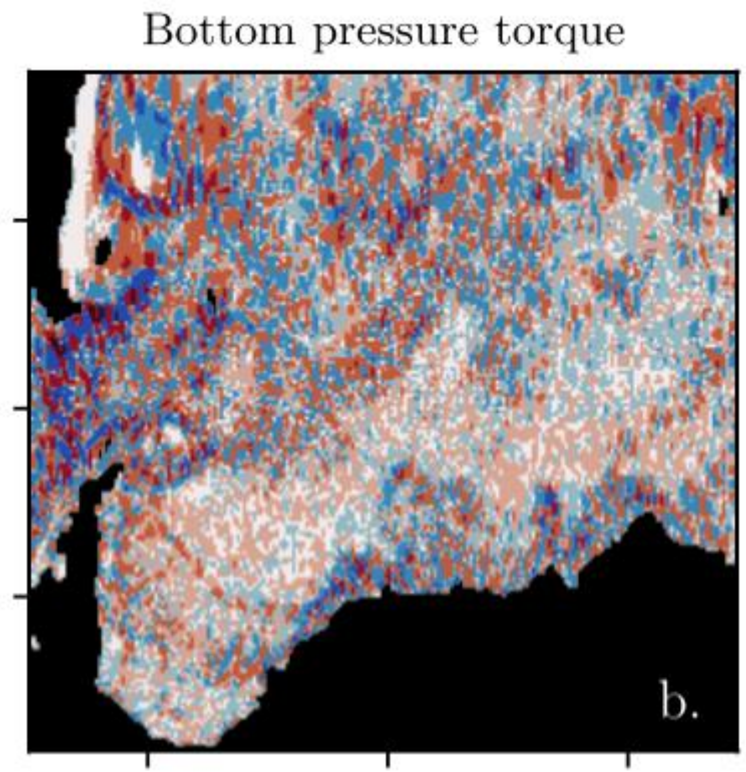
Model details

The vorticity budget of the **Weddell Gyre** was calculated in **NEMO**.

Grid point values of vorticity diagnostics are **noisy** when you have:

- Realistic **forcing**
- Realistic **geometry**

Terms in the vorticity budget are **integrated** over a **large area**

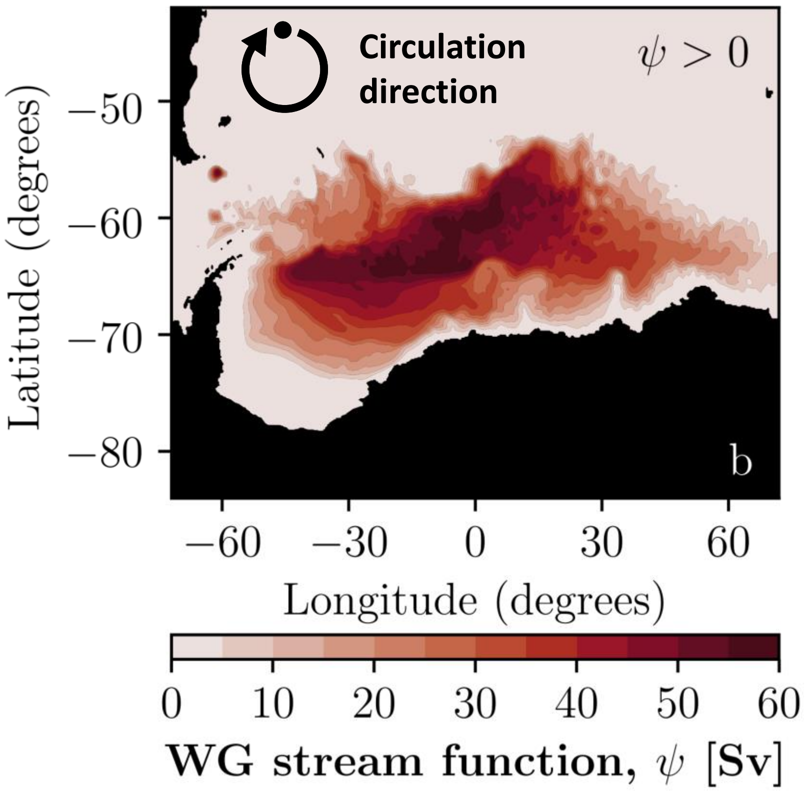


Smoothed bottom pressure torque diagnostic

Model details

Area selection alters the interpretation of the vorticity budget (Jackson et al. 2006).

Terms in the vorticity budget were integrated over the **areas enclosed** by **many streamlines**



Example contours

ψ =10 Sv ψ =40 Sv ψ =55 Sv

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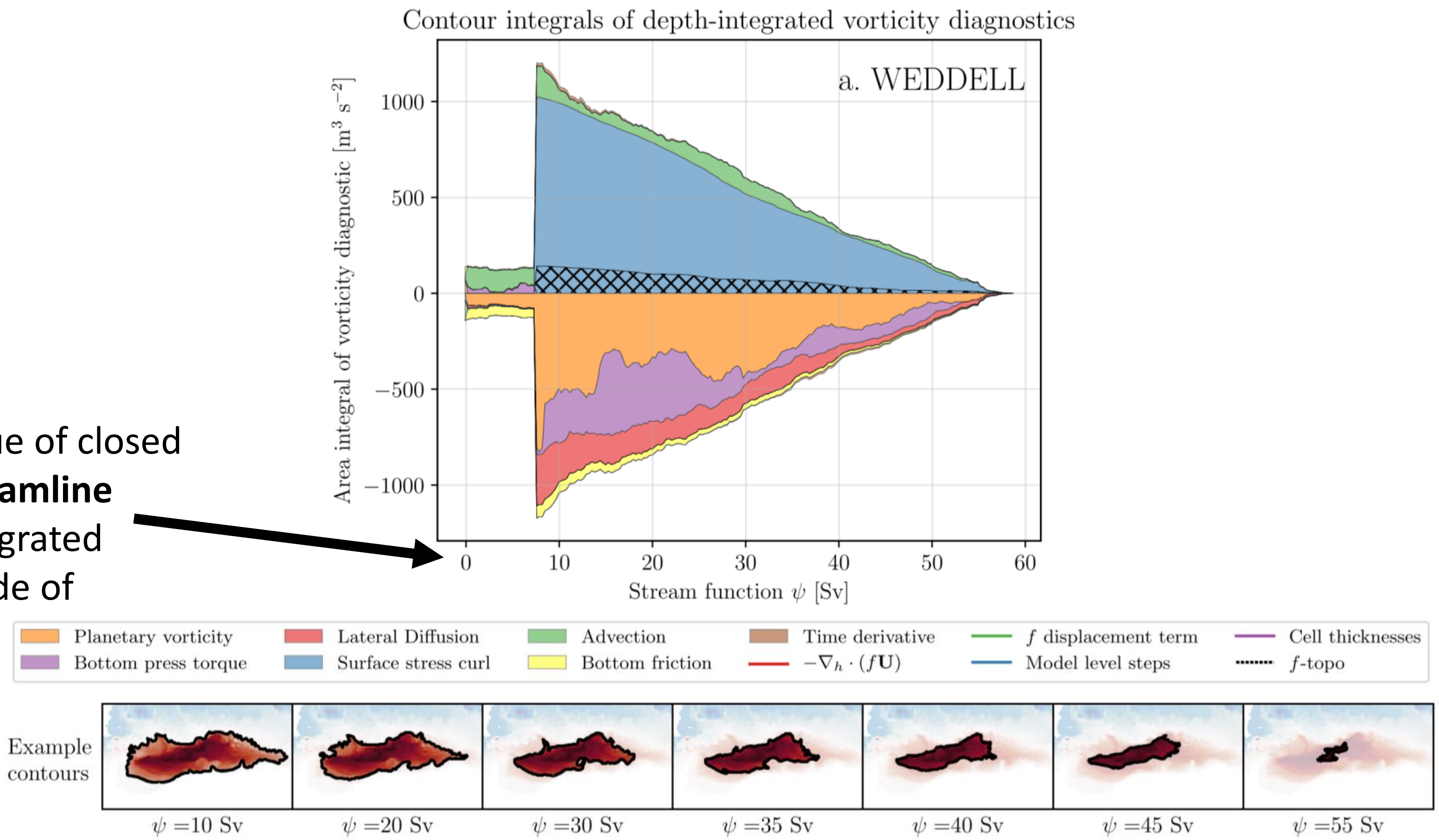
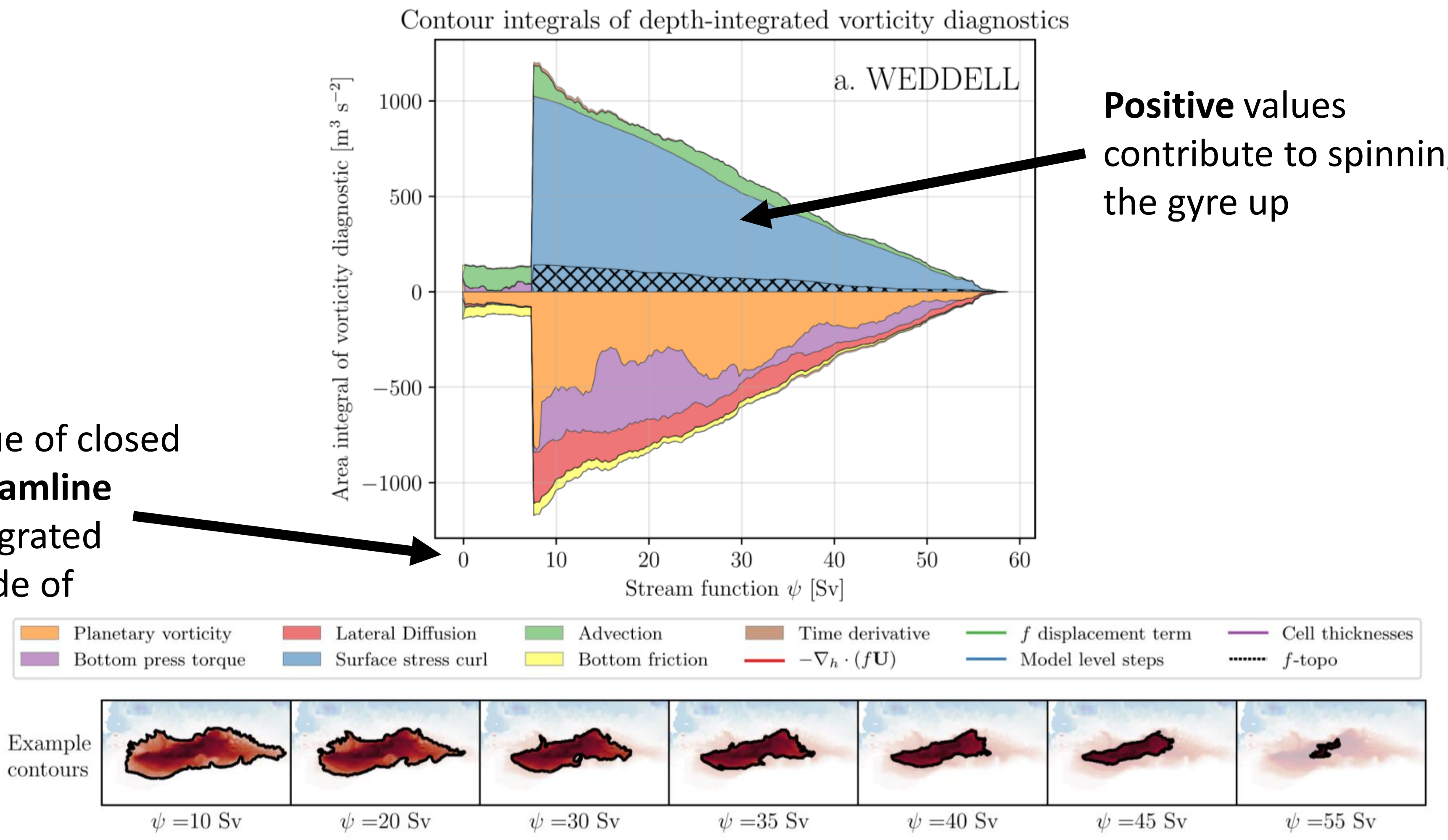
Results – Vorticity budget

Value of closed **streamline** integrated inside of



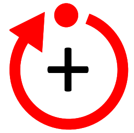
Gyre circulation

Results – Vorticity budget

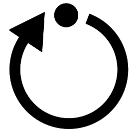


Value of closed **streamline** integrated inside of

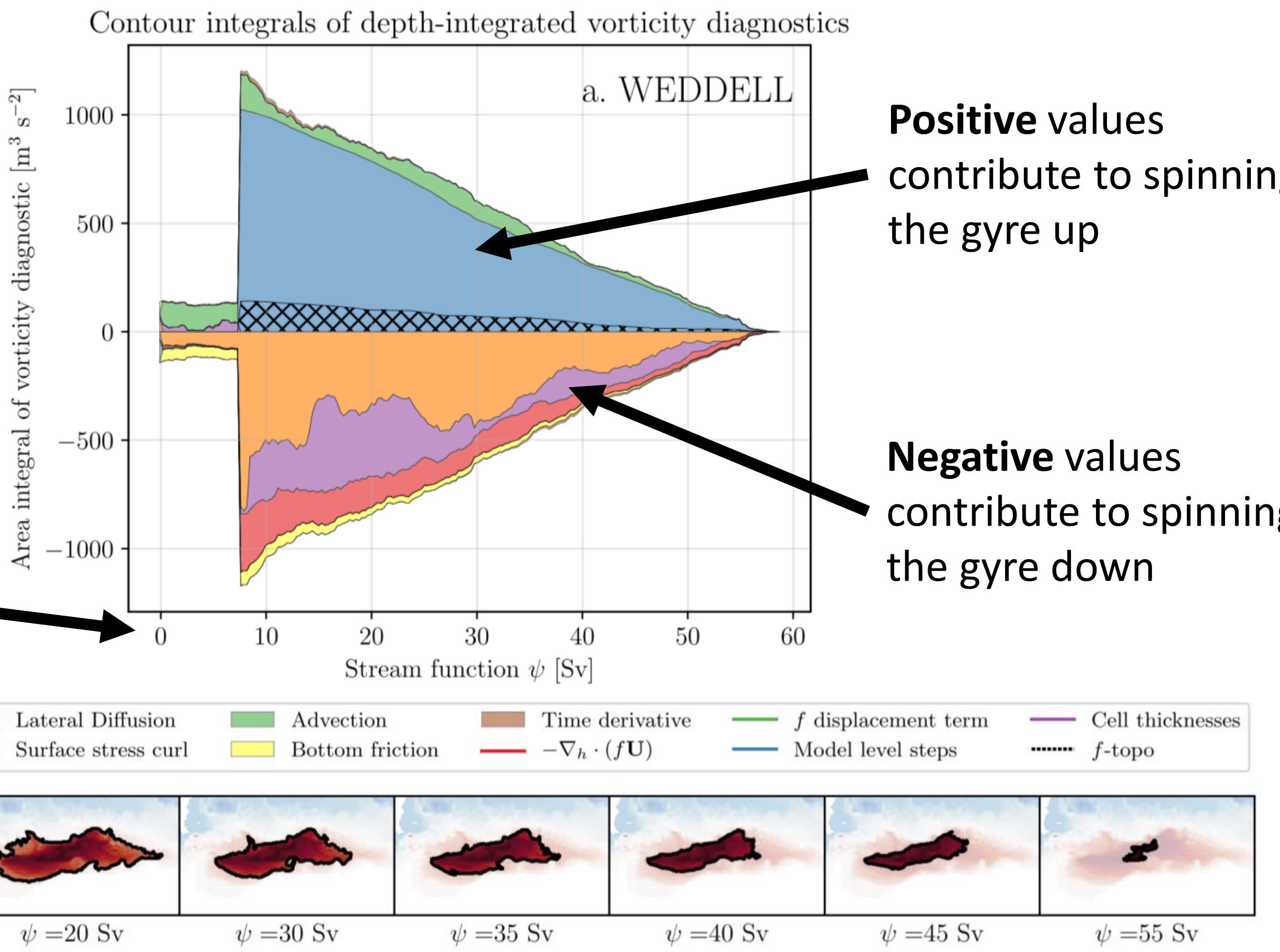
Positive values contribute to spinning the gyre up



Gyre circulation



Results – Vorticity budget



Positive values contribute to spinning the gyre up

Negative values contribute to spinning the gyre down

Value of closed **streamline** integrated inside of

Gyre circulation

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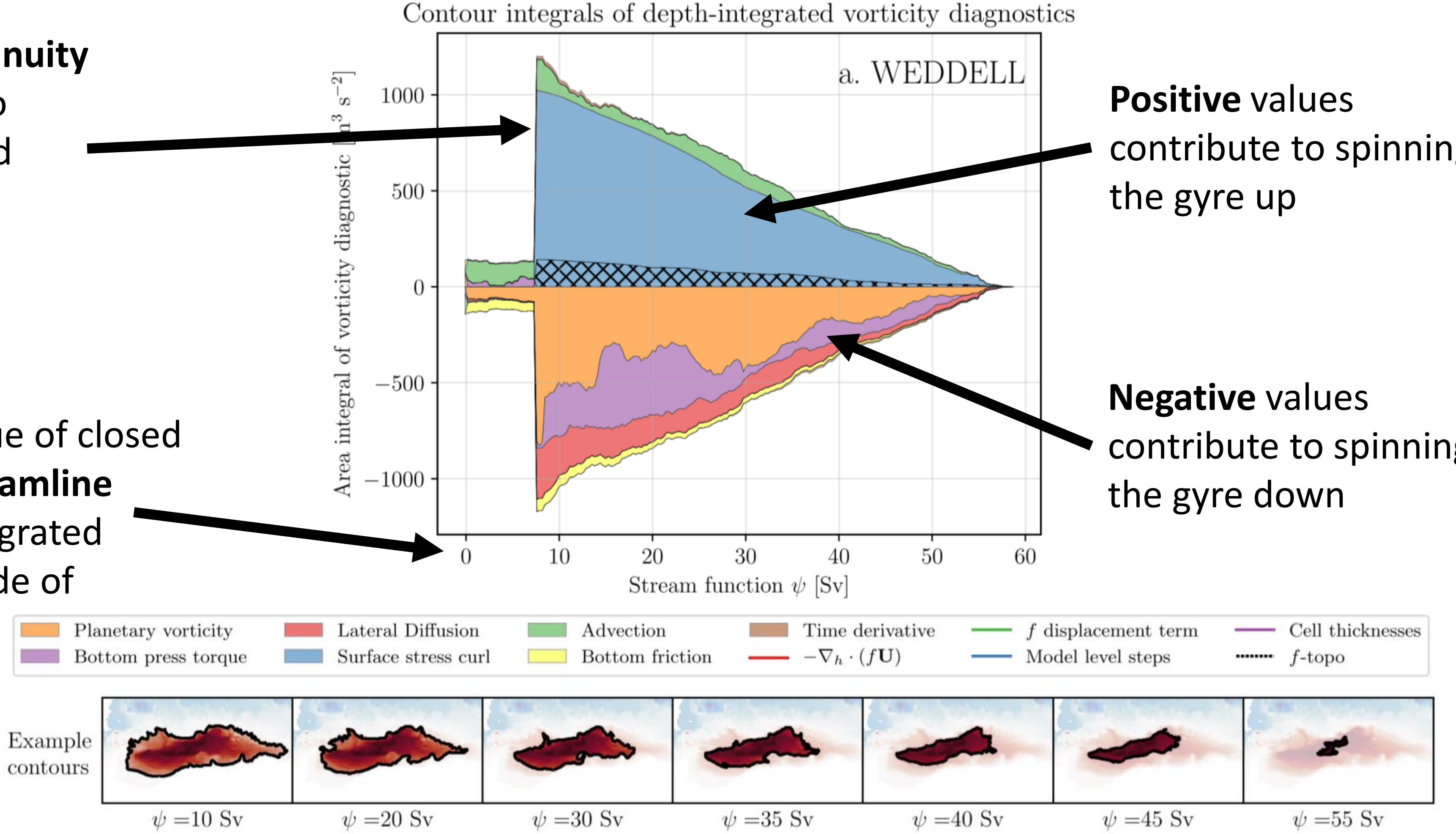
Results – Vorticity budget

Discontinuity is due to enforced contour criteria

Value of closed **streamline** integrated inside of



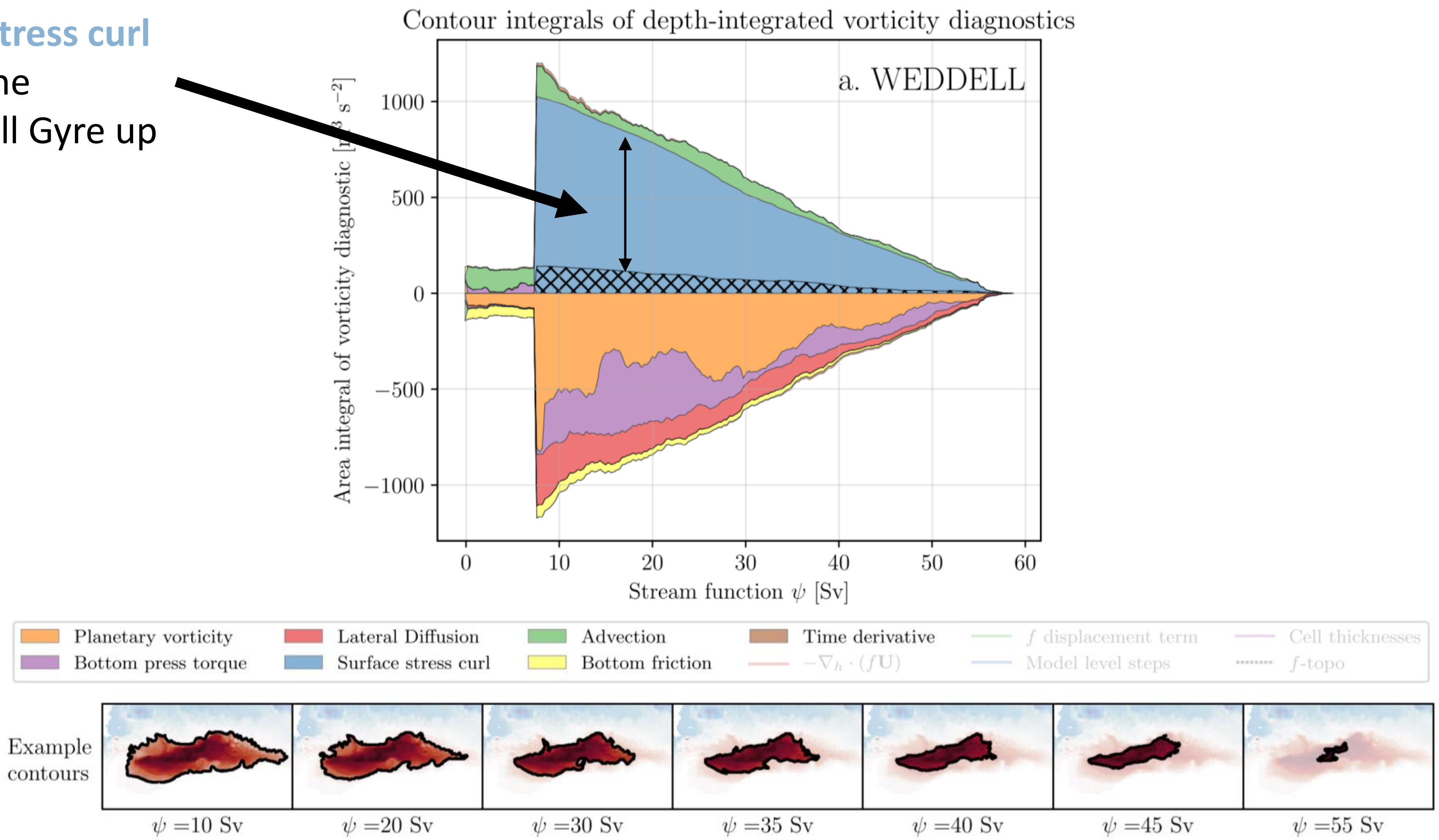
Positive values contribute to spinning the gyre up

Negative values contribute to spinning the gyre down

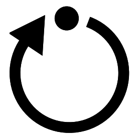
Gyre circulation

Results – Vorticity budget

Wind stress curl spins the Weddell Gyre up



Gyre circulation



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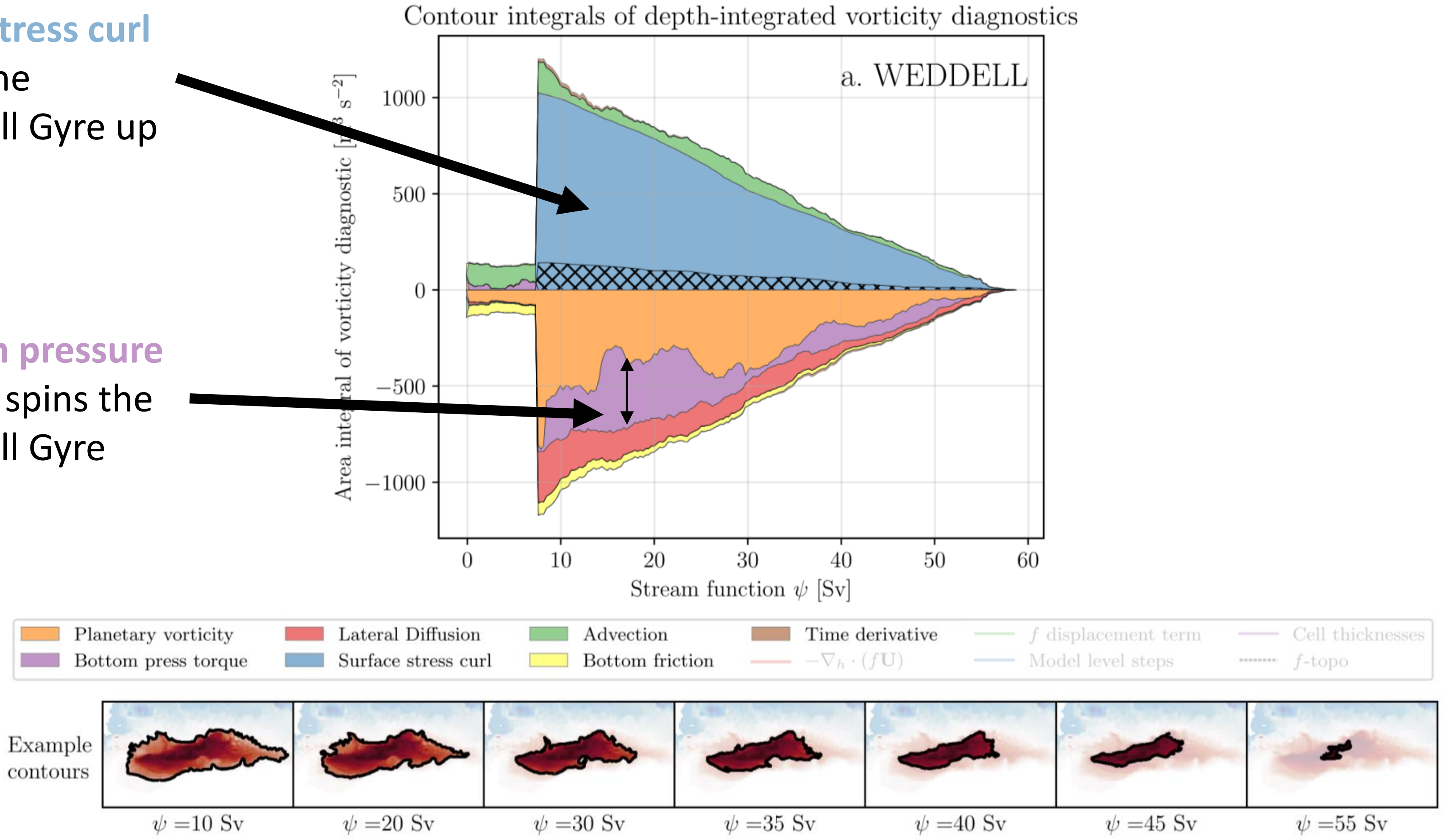
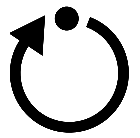
Results – Vorticity budget

Wind stress curl spins the Weddell Gyre up

Bottom pressure torque spins the Weddell Gyre down



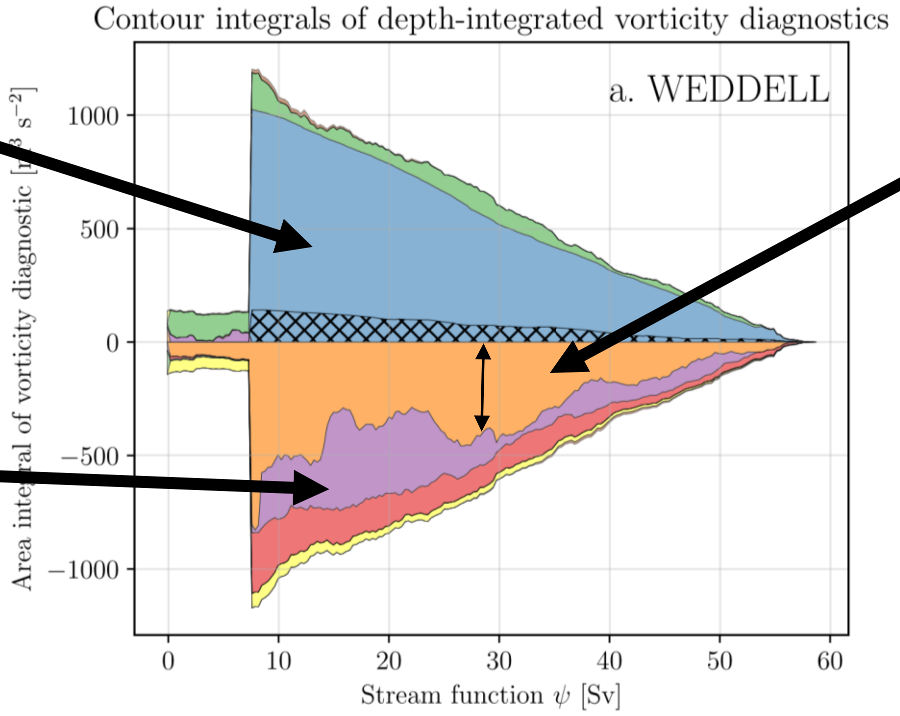
Gyre circulation

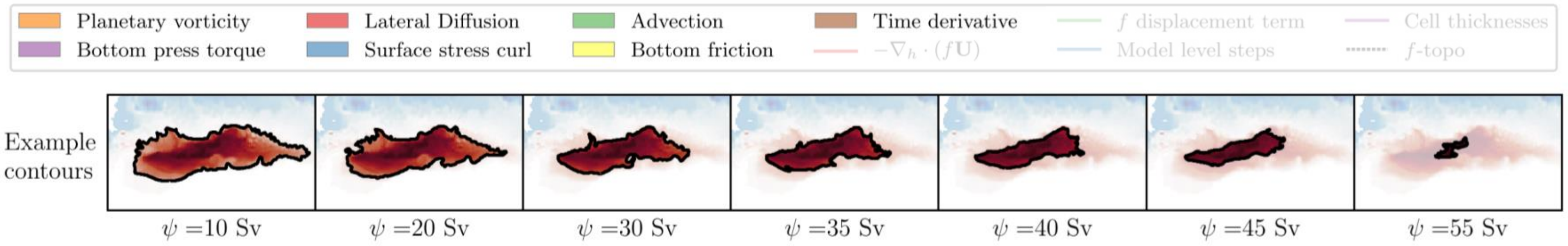


Results – Vorticity budget

Wind stress curl spins the Weddell Gyre up

Bottom pressure torque spins the Weddell Gyre down



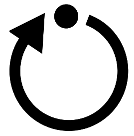


Coriolis term is the dominant opposing force?

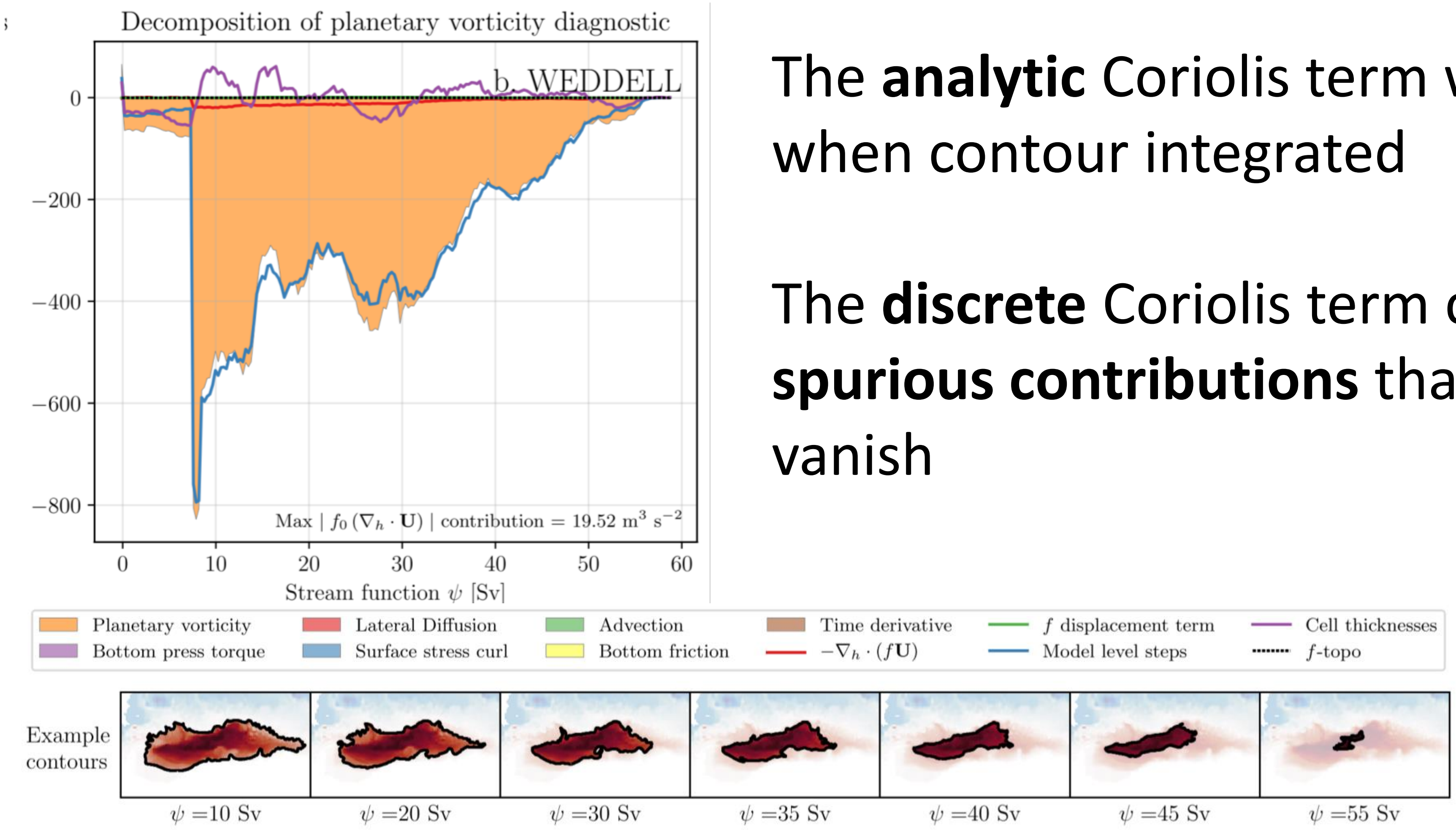
$$-\iint_{A_\psi} \nabla_h \cdot (f\boldsymbol{U})\, dA = 0$$

A consequence of the **divergence theorem.**

Gyre circulation



Results – Decomposition



The **analytic** Coriolis term will **vanish** when contour integrated

The **discrete** Coriolis term contains **spurious contributions** that do not vanish

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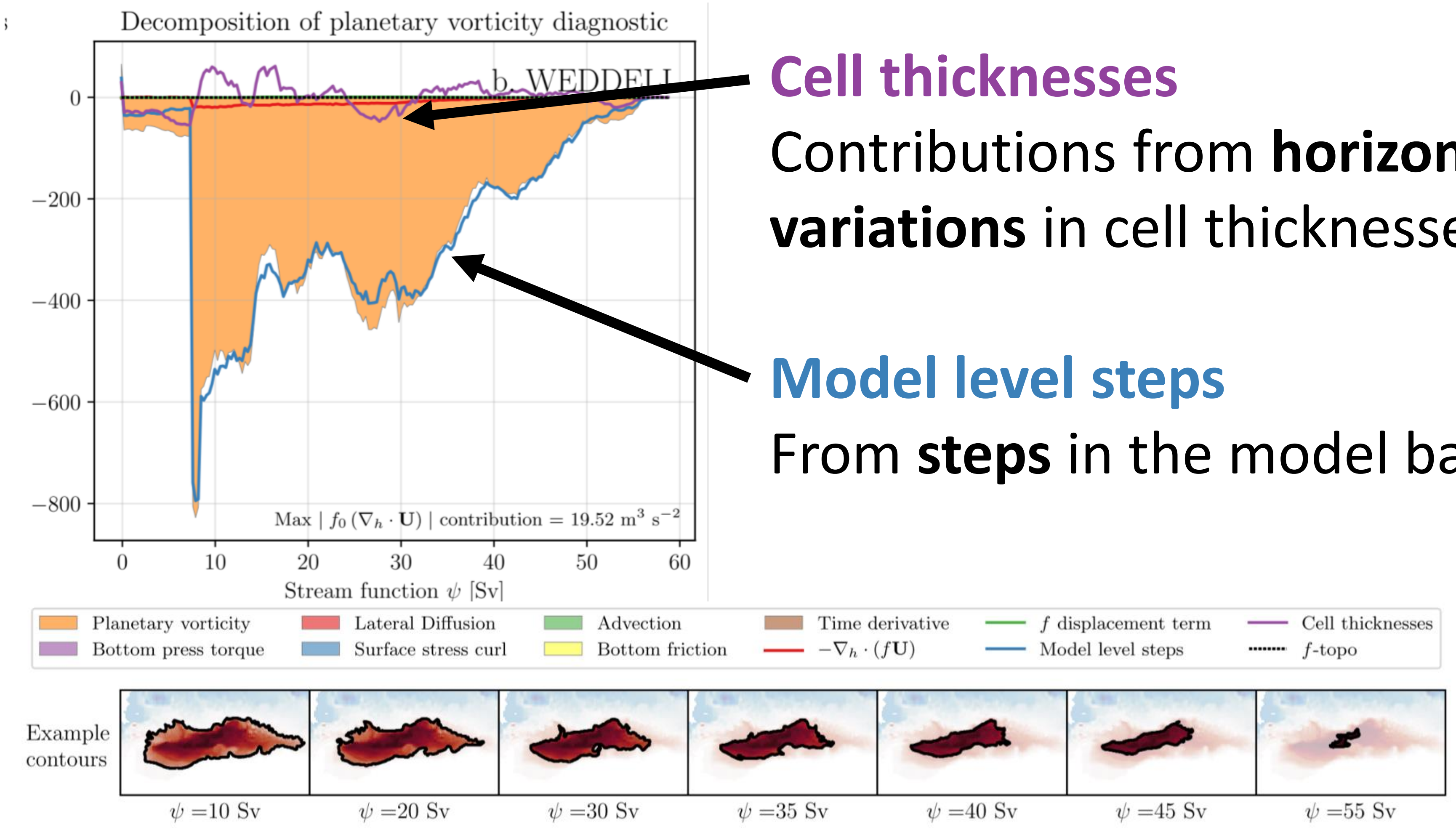
Results – Decomposition



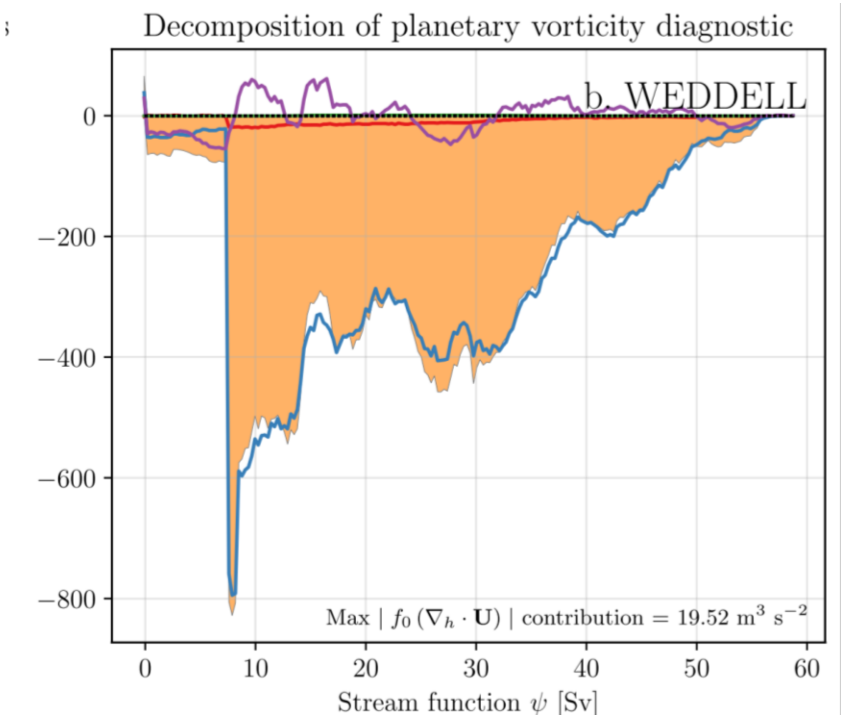
Cell thicknesses

Contributions from **horizontal variations** in cell thicknesses

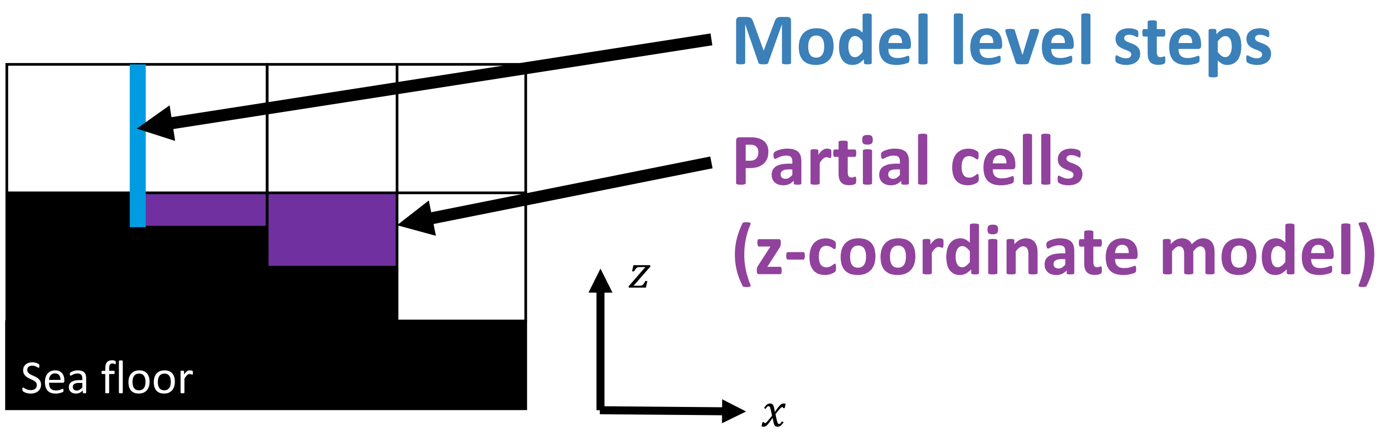
Model level steps

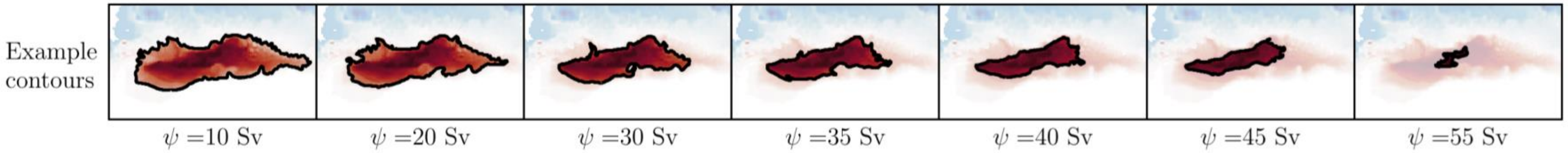
From **steps** in the model bathymetry

Results – Decomposition



Both of these contributions are **topographic**





Conclusions

- The depth-integrated vorticity budget identifies forces that **spin gyres up** and **down**
- Spurious **topographic forces** can emerge from the **discrete Coriolis acceleration**
- The identified forces emerge when using a **C-grid** with z**-coordinates**
- An alternative **horizontal or vertical discretization** (e.g. B-grid, terrain-following coordinates) will alleviate the spurious topographic forces

GitHub

afstyles/VorticityContourAnalysisForNemo

Thank you for listening

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First published: 13 May 2022 | https://doi.org/10.1029/2021MS002884

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Extra Slides

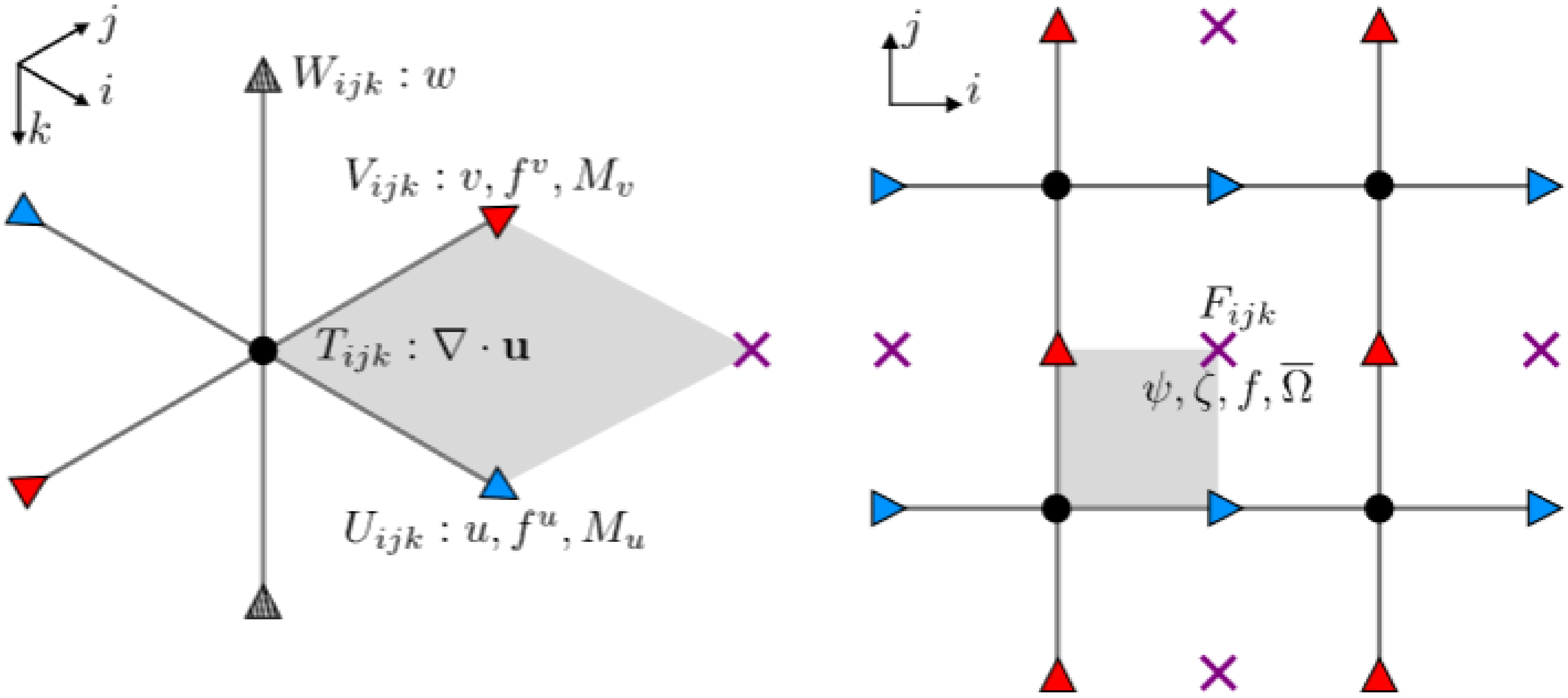


Figure 1. The distribution of variables on the C-grid in both a three dimensional (left) and horizontal (right) view. The *T*, *U*, *V*, *F*, and *W* points are shown alongside important values that are centered on these points. The *T*, *U*, *V*, and *F* points at the coordinate (*i*, *j*, *k*) lie on the four corners of the gray square. The variable *w* is the vertical velocity and M_u, M_v are the *x* and *y* components of a term in the momentum equation. Note that *k* increases downwards.

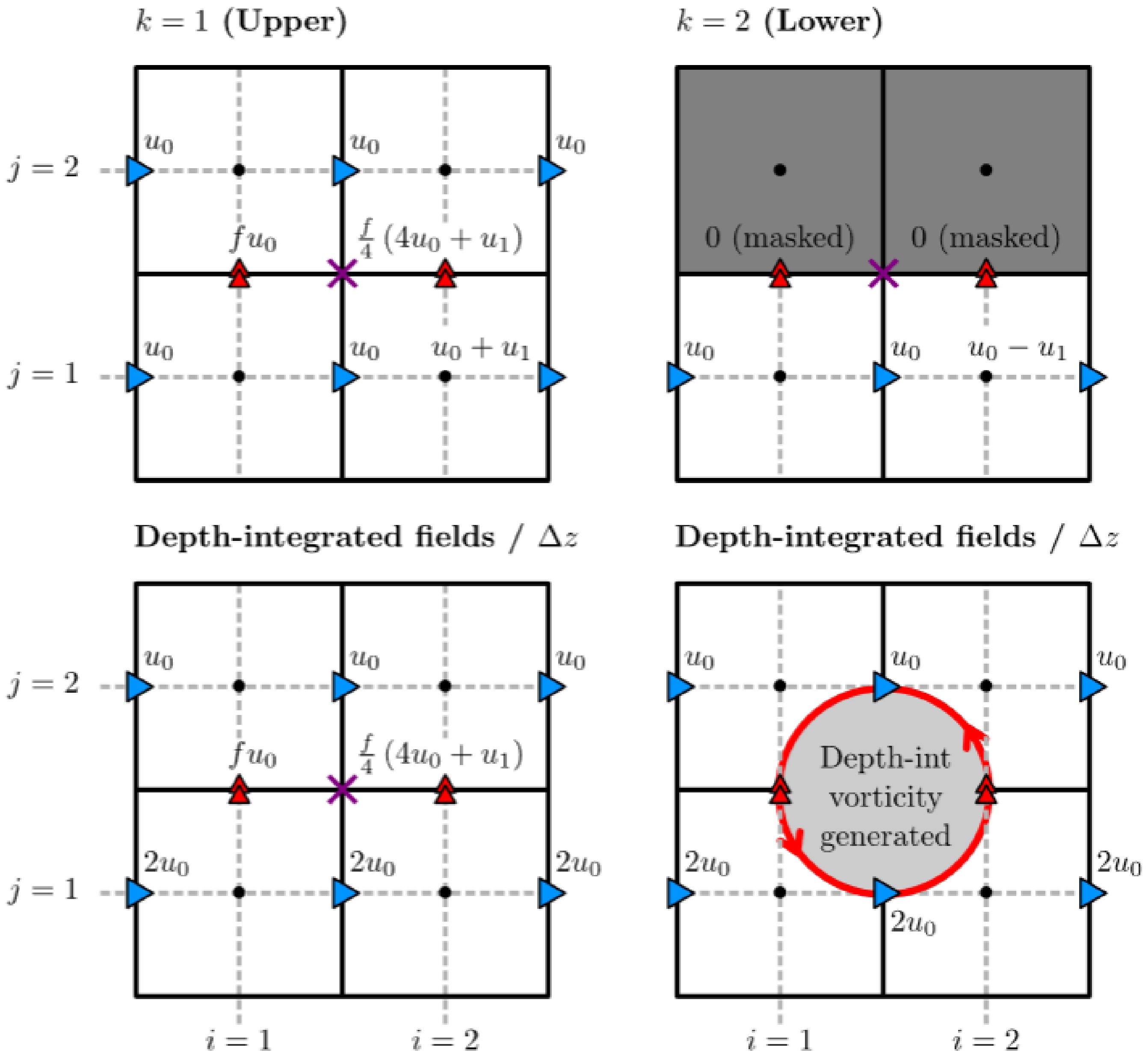
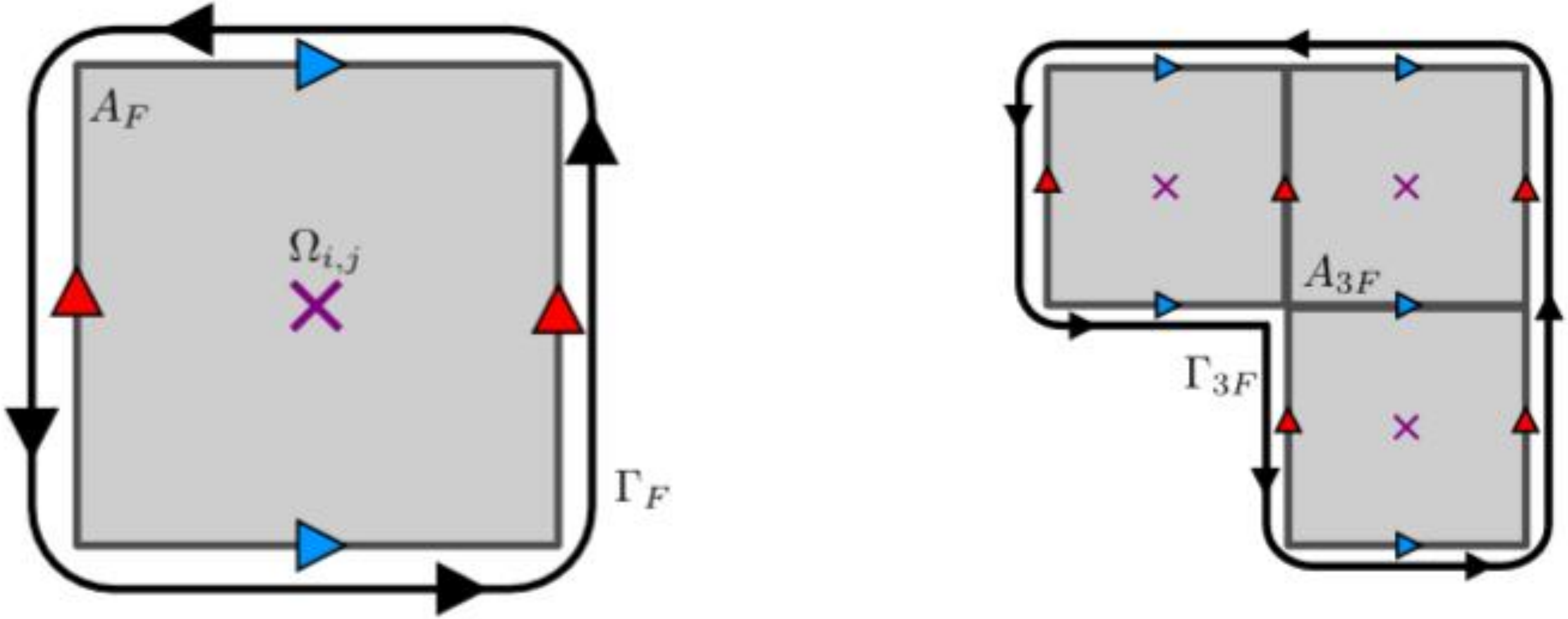


Figure 2

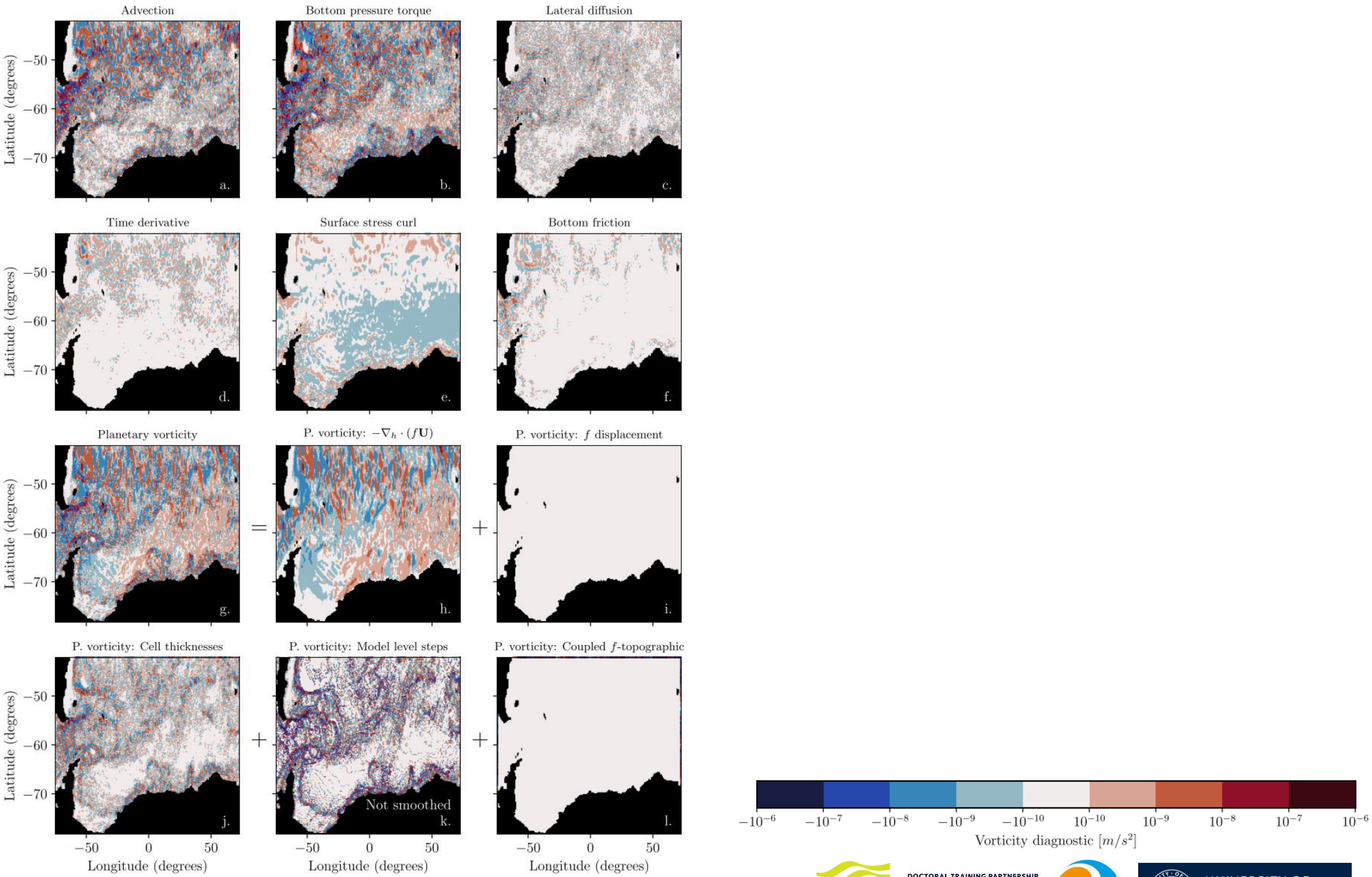
A toy model demonstrating how model levels influence the discrete Coriolis acceleration. A horizontal plan is shown for the upper and lower level as well as a view of the depth-integrated fields divided by the cell thickness Δz. Single arrows represent prescribed velocities; double arrows represent calculated Coriolis accelerations; and shaded cells represent bottom topography. Accelerations on the lower level are masked to prevent the velocity field from evolving into a flow that would violate the no penetration boundary condition. The central F point is marked by a cross and is where the depth-integrated vorticity is generated.



$$\Omega_{i,j} = \frac{1}{A_F} \iint_{A_F} \nabla \times \mathbf{M} \cdot \hat{\mathbf{k}} \, dA \qquad I_{3F} = \iint_{A_{3F}} \nabla \times \mathbf{M} \cdot \hat{\mathbf{k}} \, dA$$

$$\Omega_{i,j} = \frac{1}{A_F} \oint_{\Gamma_F} \mathbf{M} \cdot \mathbf{dl} \qquad I_{3F} = \oint_{\Gamma_{3F}} \mathbf{M} \cdot \mathbf{dl}$$

Figure 3. The application of Stokes' theorem on a C-grid. The vorticity diagnostic Ω is equivalent to the normalized line integral of $\mathbf{M}$ around a single F cell of area A_F. The area integral of Ω over a collection of F cells (e.g., A_{3F}) is equivalent to the line integral of $\mathbf{M}$ along the perimeter (e.g., Γ_{3F}).



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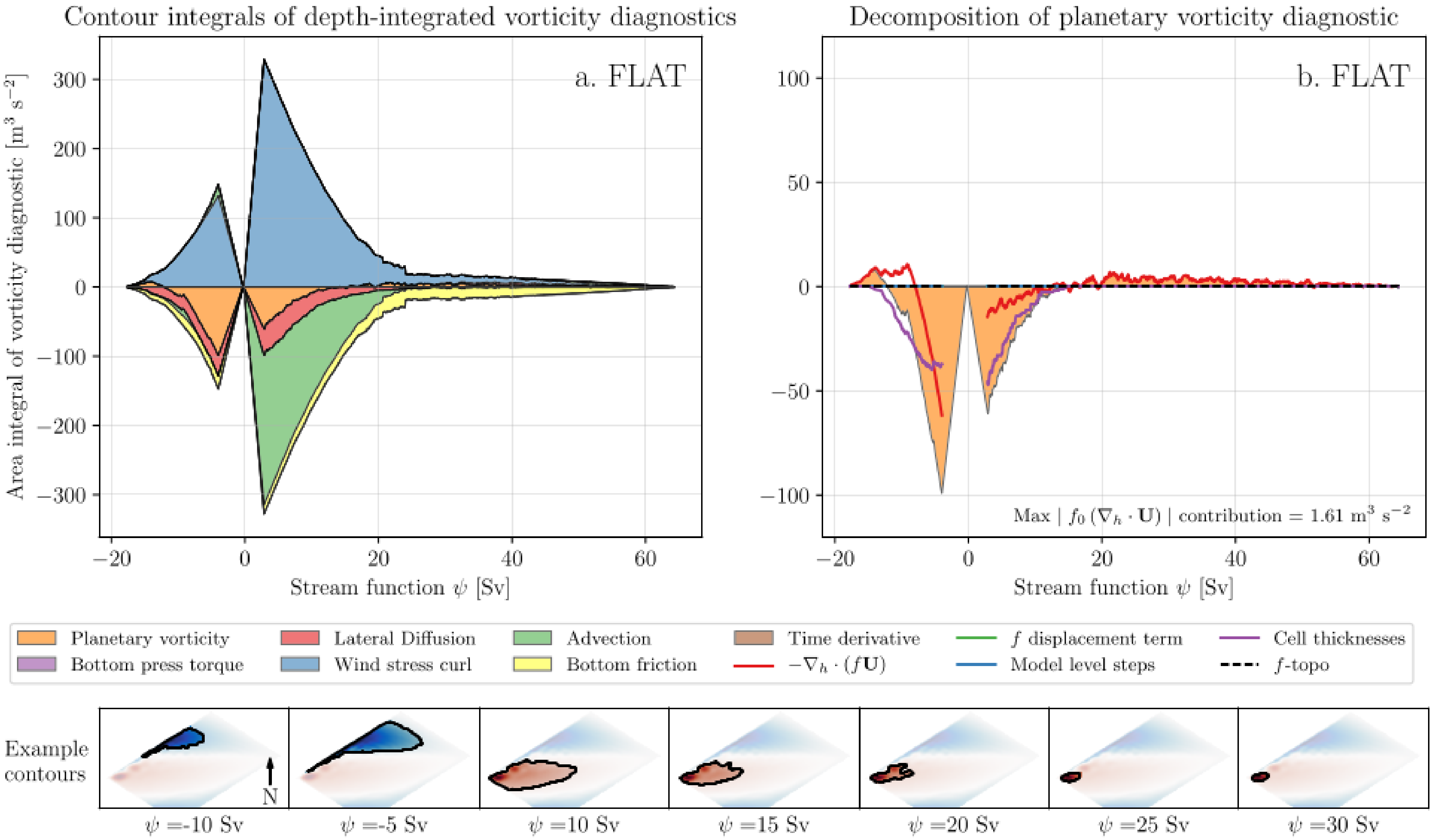
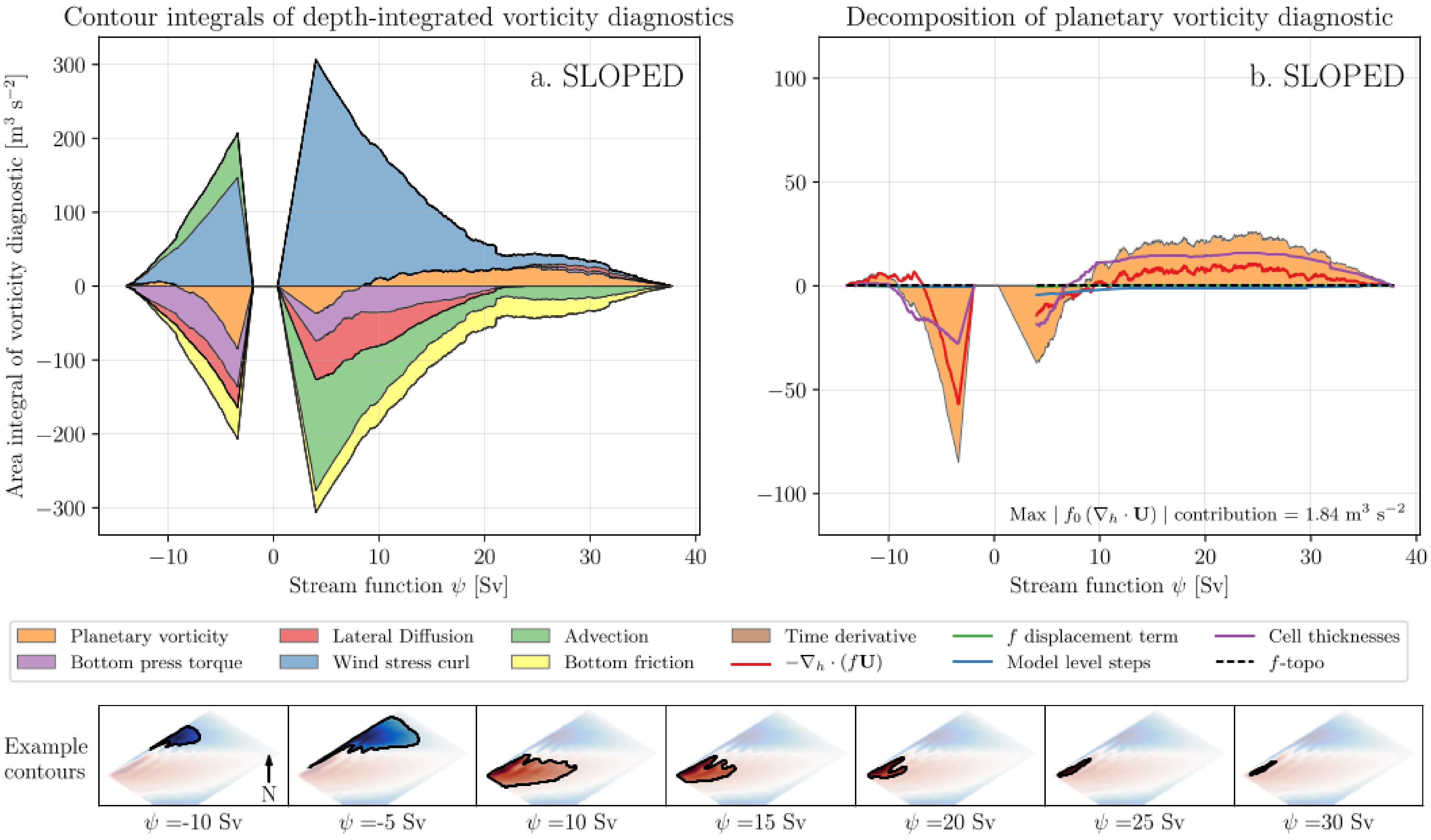


Contour integrals of depth-integrated vorticity diagnostics
Decomposition of planetary vorticity diagnostic
a. FLAT
b. FLAT
Area integral of vorticity diagnostic [m^3 s^{-2}]
Stream function ψ [Sv]
Max | f_0 ($\nabla_h \cdot$ U) | contribution = 1.61 m^3 s^{-2}
Planetary vorticity
Bottom press torque
Lateral Diffusion
Wind stress curl
Advection
Bottom friction
Time derivative
$-\nabla_h \cdot (f\mathbf{U})$
f displacement term
Model level steps
Cell thicknesses
f-topo
Example contours
N
ψ =-10 Sv
ψ =-5 Sv
ψ =10 Sv
ψ =15 Sv
ψ =20 Sv
ψ =25 Sv
ψ =30 Sv



Contour integrals of depth-integrated vorticity diagnostics
a. SLOPED
Area integral of vorticity diagnostic [m^3 s^{-2}]
300
200
100
0
−100
−200
−300
−10
0
10
20
30
40
Stream function ψ [Sv]
Decomposition of planetary vorticity diagnostic
b. SLOPED
100
50
0
−50
−100
Max | $f_0\,(\nabla_h \cdot \mathbf{U})$ | contribution = 1.84 m^3 s^{-2}
−10
0
10
20
30
40
Stream function ψ [Sv]
Planetary vorticity
Bottom press torque
Lateral Diffusion
Wind stress curl
Advection
Bottom friction
Time derivative
$-\nabla_h \cdot (f\mathbf{U})$
f displacement term
Model level steps
Cell thicknesses
f-topo
Example contours
N
ψ =-10 Sv
ψ =-5 Sv
ψ =10 Sv
ψ =15 Sv
ψ =20 Sv
ψ =25 Sv
ψ =30 Sv